

**ENTRANCE EXAMINATION IN MATHEMATICS FOR APPLICANTS ENTERING  
MASTER'S PROGRAMS OF PHYSTECH SCHOOL OF RADIO ENGINEERING AND  
COMPUTER TECHNOLOGY**

**Form:** oral with writing assignment

**Duration of writing part:** 45 min

**Duration of oral part:** up to 45 min. Discussion of writing part results and interview on theoretical questions

**Exam rules:** During preparation of writing assignment it is allowed to use printed textbooks, papers, printed and online handbooks. During oral answer all sources, except of self-written, are prohibited. All interactive sources are strongly prohibited including searching systems, chats and others.

**A. Theoretical questions.**

*Please describe and explain:*

1. Geometry. How to find an area of figures? Get examples for triangle, circle, trapeze.
2. Cosines theorem (Law of cosines). Explain and get examples.
3. Trigonometry. Explain trigonometric function: sin, cos, tg. Known relationship between them. Get examples.
4. Explain inverse trigonometric function: arcsin, arccos, arctg. Get examples.
5. What is Matrix-vector equation, how we can solve matrix-vector equations? Describe examples of direct and iterative methods for solving matrix equations.
6. What is the Matrix operation? Explain matrix product, matrix power, matrix inverse, identity matrix. etc.
7. What is the derivative? How to find derivatives in analytical way (examples). How to find derivatives in computational way.
8. Chain rule for finding derivatives. Explain and get examples.
9. What is the integral? How to find integrals in analytical way (examples). How to find integrals in computational way.
10. What is the complex number? Imaginary one? Explain operations with complex numbers, addition, multiplication, conjugation etc.
11. What is the optimization? Explain how to find optimum of a function, get examples.
12. Gradient descent method, explain how it works.
13. What is the probability? Get examples of probability distribution functions.
14. How to estimate probability in computational way? Get examples.

**Writing assignment.**

Explain your solution step-wise.

1. Find partial derivatives of function  $z = x \ln y + \frac{y}{x}$
2. Take an integral  $\int \frac{1}{x^3} dx =$
3. Find differential of function  $z = x \sin(y)$  at point  $P(-1; -\pi/2)$
4. Solve matrix equation  $\begin{pmatrix} 3 & 7 \\ 2 & 8 \end{pmatrix} \cdot X = \begin{pmatrix} 4 & 8 \\ 6 & 2 \end{pmatrix}$

5. Rotate the vectors

$$\mathbf{x}_1 := \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \quad \mathbf{x}_2 := \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

by  $30^\circ$ .

6. Compute the derivative  $f'(x)$  of the logistic sigmoid

$$f(x) = \frac{1}{1 + \exp(-x)}.$$

Simplify answer by using  $f(x)$

### **Recommended literature**

1. M. P. Deisenroth, A. A. Faisal, C. S. Ong, Mathematics for Machine Learning \ \ To be published by Cambridge University Press. 2020, Available at <https://mml-book.github.io/>
2. G. Strang. Calculus \ \ MA: Wellesley College (1991). Available at <https://ocw.mit.edu/resources/res-18-001-calculus-online-textbook-spring-2005/textbook/>